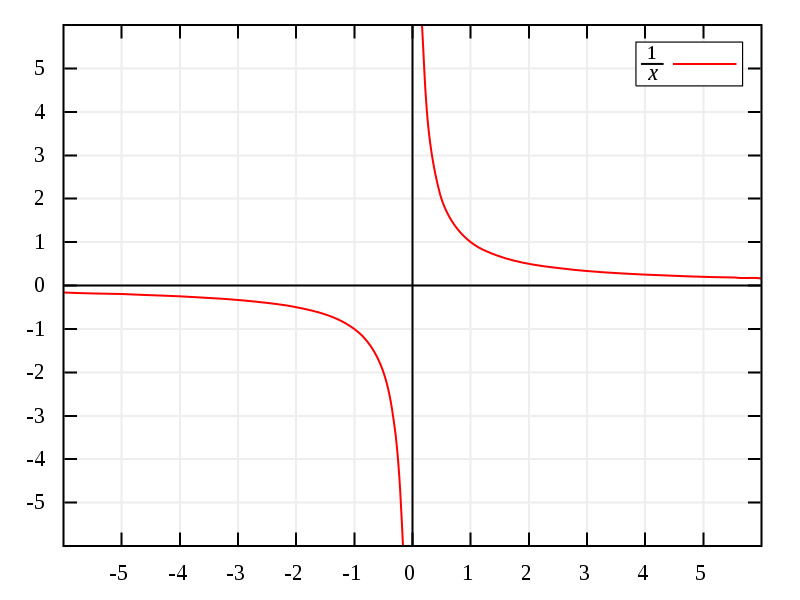
**Asymptote** – a line where a function approaches one value as it continues for infinity.

Graph 1

**Horizontal asymptote** – where the approaches having some **constant value** when approaches infinity. (Graph 1)

**Vertical asymptote** – where the approaches having some **constant value** when approaches infinity. (Graph 1)

# Infinite limits

Some limits **become infinite** when evaluated at certain points.

“The limit of becomes infinity as approaches .”

“As approaches , increases without bound.”

Note: infinity () is **not a real number**, just a way to express the idea of the limit not having a real value.

Limits can also be **negative infinity**.

Wherever the **limit** of a function is **infinity** (or negative infinity) there is a **vertical asymptote**.

Limits can be one-sided with **different values on each side**; vertical asymptotes can be positive infinity at one side and negative infinity at the other side.

Consider Graph 1 and find vertical asymptotes – these are infinite limits.

For all odd integers : or .

For all even integers : .

# Limits at Infinity

Some limits are evaluated where **x approaches infinity** (or negative infinity).

“The limit of is as approaches infinity.”

“As increases without bound, the limit of is .”

Wherever the **limit** of a function converges as  **approaches** **infinity** (or negative infinity) there is a **horizontal asymptote**.

Infinity can only be approached from the left, and negative infinity can only be approached form the left.

Consider Graph 1 and find horizontal asymptotes – these are limits at infinity.

For all integers : and

# Infinite Limits at Infinity

Some **limits equal infinity as x approaches infinity** (or negative infinity).

Graph 2

“The limit of is infinity as approaches infinity.”

“As increases without bound, the limit of increases without bound.”

Infinity can only be approached from the left, and negative infinity can only be approached form the left.

Consider Graph 2 and notice where approaches infinity as approaches infinity.

For all odd integers : and .

For all even integers : and .

# Precise Definitions of Limits

Graph 3

## Infinite Limits

For a function that is defined on some interval,

If , then,

For every positive number there is a number such that,

If , then, .

See Graph 3.

## Limit at Infinity

Graph 4

For a function defined on the interval ,

If , then,

For every number , there is a number such that,

If , then, .

See Graph 4.